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# HOW LINEAR PROGRAMMING SOLVES A SURPRISING VARIETY OF CONSUMER RESEARCH PROBLEMS

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Video and Additional Resources: <https://bit.ly/36faZPQ>



**AIGORA**



**PEPSICO**

# Background and Insight

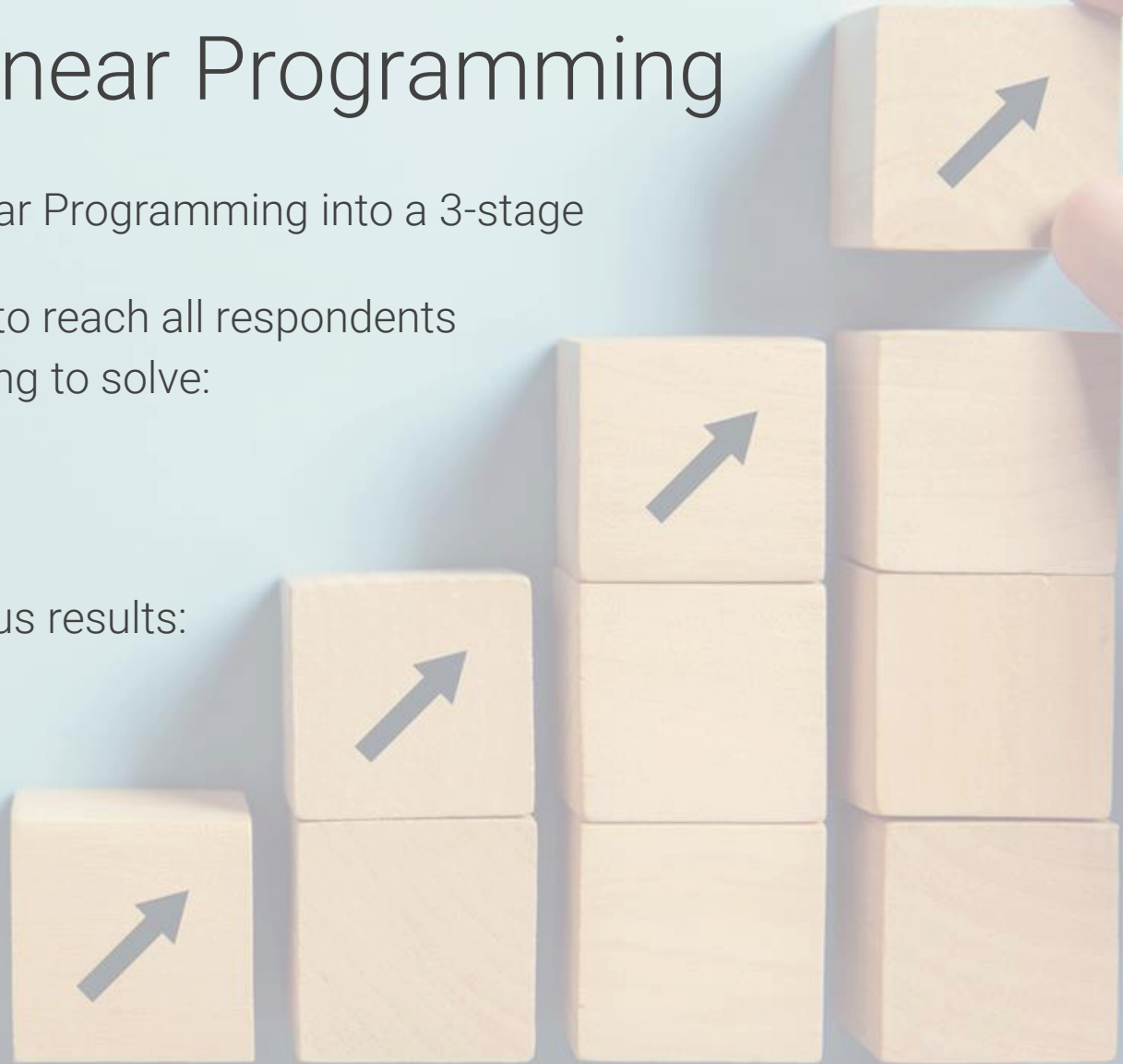
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- **Linear programming** is a tool from operations research that rapidly optimizes linear problems
- Daniel Serra (2013) had key insight that TURF problems can be made linear
  - To implement TURF via linear programming:
    - Record in a binary indicator variable  $r_{ij}$  whether item  $j$  reaches subject  $i$
    - Create a binary indicator variable  $p_j$  for each item that expresses whether the item is selected by the chosen portfolio
    - Create a binary indicator variable  $s_i$  for each subject that expresses whether the subject is selected by the chosen portfolio
    - Create a constraint  $s_i \leq \sum_j r_{ij} * p_j$  for each subject to express that, if no item that reaches subject  $i$  is selected, then subject  $i$  is not reached
  - All TURF-related quantities can be expressed using these variables:
    - Reach =  $\sum_i s_i$  (*total number of respondents reached*)
    - Frequency =  $\sum_j \sum_i r_{ij} * p_j$  (*total of selected item reaches*)
    - Penetration =  $(\sum_j (\sum_i r_{ij} * p_j)^{-1})^{-1}$  (*harmonic mean of selected item reaches*)
  - Linear programming of TURF can then be conducted using the open source **lpSolve** package in R
- Ennis and Russ (2016, 2018) extended Serra's insight:
  - Staged approach with eTURF 2.0
  - Pareto efficient solution with Comprehensive Market Coverage Analysis (CMCA)

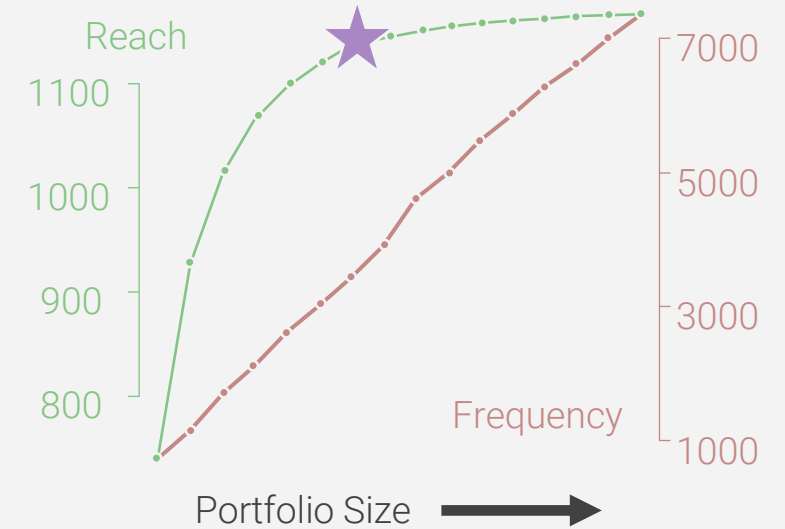
# eTURF 2.0 as Linear Programming

- **Ennis and Russ (2016)** systematized the use of Linear Programming into a 3-stage process called **eTURF 2.0**
- First determine  $P$ , fewest number of items required to reach all respondents
- For each portfolio size  $p \leq P$ , use linear programming to solve:
  - Stage A: Maximize reach
  - Stage B: Maximize frequency
  - Stage C: Maximize penetration
- The constraints for each stage are based on previous results:
  - Stage A:  
Portfolio size =  $p$
  - Stage B:  
Portfolio size =  $p$   
Reach = Max reach found in A
  - Stage C:  
Portfolio size =  $p$   
Reach = Max reach found in A  
Frequency = Max frequency found in B
- Using eTURF 2.0, we find optimal portfolios for every portfolio size up to full reach



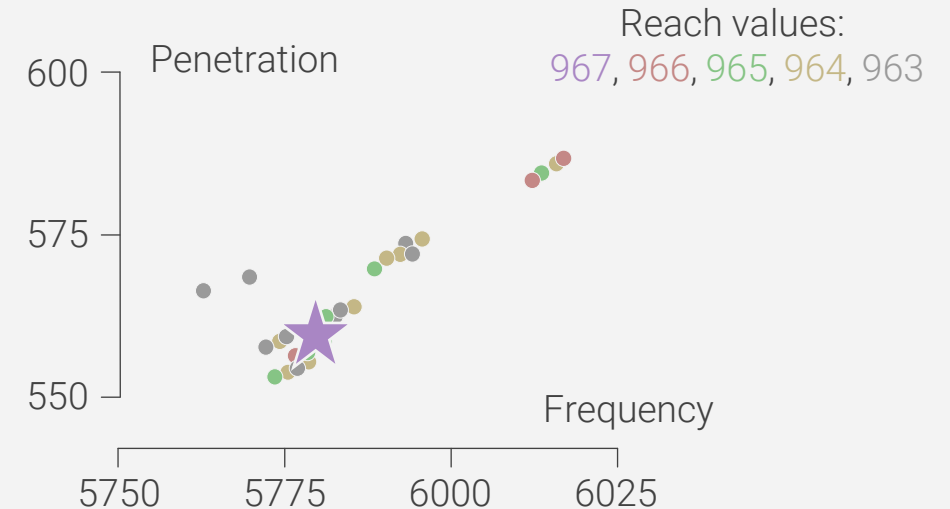
# Scenarios

- 1200 respondents provided purchase interest ratings online for **30 flavor concepts** on a 4-point scale:
- Steps to a recommended portfolio:
  - Determine an upper bound on number of concepts needed to reach all 1167 reachable respondents
  - Find an optimal portfolio for every portfolio size up to the bound
  - Conduct statistical testing to determine recommended size
- Size 7 not significantly different from size 8, so **size 7** recommended



- 1000 respondents evaluate **30 flavor concepts**
  - Can reach all 967 reachable with 10 concepts
- Constrain reach to be at least 963 (99.5% of maximum)
- 633 total solutions for 10 concepts (a subset shown)
- Conduct similar analyses for all smaller portfolios to decide recommendation

Many solutions outperform the **optimal reach solution** on both frequency and penetration.



# Future Directions and Conclusion

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WHAT'S  
NEXT?

- **Future Directions:**
  - Respondents can be assigned different weights so that, for example, portfolios can be designed to disproportionately appeal to heavier users of a product category
  - Custom details can be programmed into the search, such as a desire to include certain items or combinations of items
  - Restrictions can be placed on the search to reflect real-world restrictions, such as "If item A is selected, then don't select item B"
- **Conclusion:**
  - Many market research problems can be recast as linear problems
  - Once problems are seen as linear they can be solved using linear programming
  - The speed and scale of linear programming means that large families of problems can be considered, leading to global insights otherwise undiscoverable
- **Additional Resources:**

Please visit <https://bit.ly/346rdbc> for:

  - References
  - Link to video
  - Contact information
  - Code used in this poster
  - Q&A follow up
  - And more!

